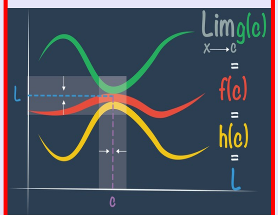
Calculus I

Lecture 11



Feb 19-8:47 AM

Class Quiz 10

Show that the equation $x^4 + x - 3 = 0$ has at least one real Solution on the interval $[1, 2]$. $\boxed{\text{Be very detailed.}}$

Let $f(x) = x^4 + x - 3$

Polynomial Function $\Rightarrow$ Continuous on $(-\infty, \infty)$

$f(1) = 1^4 + 1 - 3 = -1$

Solution $f(x) = 0$

$f(2) = 2^4 + 2 - 3 = 16 + 2 - 3 = 15$

By I.V.T., there is a number c in $(1, 2)$ such that $f(c) = 0 \Rightarrow f(1) < f(c) < f(2)$

$-$ 0 $+$

Mar 19-8:15 AM

Evaluate

1) $\lim_{x\to\infty} \frac{3x-2}{2x+3} = \frac{\infty}{\infty}$ I.F.

$= \lim_{x\to\infty} \frac{\frac{3x}{x} - \frac{2}{x}}{\frac{2x}{x} + \frac{3}{x}} = \lim_{x\to\infty} \frac{3 - \frac{2}{x}\to 0}{2 + \frac{3}{x}\to 0} = \boxed{\frac{3}{2}}$

2) $\lim_{x\to\infty} \frac{x - x\sqrt{x}}{2x\sqrt{x} + 3x - 5} = \frac{-\infty}{\infty}$ I.F.

Recall
$x\sqrt{x} = x \cdot x^{\frac{1}{2}}$
$= x^{1+\frac{1}{2}}$
$= x^{3/2}$

$= \lim_{x\to\infty} \frac{x - x^{3/2}}{2x^{3/2} + 3x - 5}$

$= \lim_{x\to\infty} \frac{\frac{x}{x^{3/2}} - \frac{x^{3/2}}{x^{3/2}}}{\frac{2x^{3/2}}{x^{3/2}} + \frac{3x}{x^{3/2}} - \frac{5}{x^{3/2}}} = \lim_{x\to\infty} \frac{\frac{1}{\sqrt{x}}\to 0 - 1}{2 + \frac{3}{\sqrt{x}}\to 0 - \frac{5}{x^{3/2}}\to 0}$

$= \boxed{\frac{-1}{2}}$

Mar 19-9:06 AM

$\lim_{x\to\infty} \frac{\sqrt{9x^6 - x}}{x^3 + 1} = \frac{\infty}{\infty}$ I.F.

$\sqrt[2]{x^6} = x^{\frac{6}{2}} = x^3$

$x^3 = \sqrt{x^6}$

As $x \to \infty$

$= \lim_{x\to\infty} \frac{\frac{\sqrt{9x^6 - x}}{x^3}}{\frac{x^3 + 1}{x^3}} = \lim_{x\to\infty} \frac{\sqrt{\frac{9x^6 - x}{x^6}}}{\frac{x^3 + 1}{x^3}}$

$= \lim_{x\to\infty} \frac{\sqrt{\frac{9x^6}{x^6} - \frac{x}{x^6}}}{\frac{x^3}{x^3} + \frac{1}{x^3}} = \lim_{x\to\infty} \frac{\sqrt{9 - \frac{1}{x^5}\to 0}}{1 + \frac{1}{x^3}\to 0} = \frac{\sqrt{9}}{1} = \boxed{3}$

Mar 19-9:18 AM

Evaluate $\lim_{x\to-\infty} \frac{\sqrt{9x^6-x}}{x^3+1} = \frac{\infty}{-\infty}$ I.F.

As $x\to-\infty$

$x^3 = -\sqrt{x^6}$

$$\lim_{x\to-\infty} \frac{-\sqrt{\frac{9x^6-x}{x^6}}}{\frac{x^3+1}{x^3}} = \lim_{x\to-\infty} \frac{-\sqrt{9-\cancel{\frac{1}{x^5}}^{\to 0}}}{1+\cancel{\frac{1}{x^3}}^{\to 0}} = \frac{-\sqrt{9}}{1} = \boxed{-3}$$

Mar 19-9:26 AM

Evaluate $\lim_{x\to\infty} \sqrt{x}\,\sin\frac{1}{x} = \infty\cdot 0$ I.F.

$$\lim_{x\to\infty}\sqrt{x}\cdot\sin\frac{1}{x} = \lim_{x\to\infty}\frac{\sin\frac{1}{x}}{\frac{1}{\sqrt{x}}}$$

Recall $a\cdot b = \frac{b}{\frac{1}{a}}$

$$= \lim_{x\to\infty}\frac{\sin\frac{1}{x}}{\frac{\sqrt{x}}{x}} = \lim_{x\to\infty}\frac{\sin\frac{1}{x}}{\sqrt{x}\cdot\frac{1}{x}}$$

$$= \lim_{x\to\infty}\frac{1}{\sqrt{x}} \cdot \lim_{x\to\infty}\frac{\sin\frac{1}{x}}{\frac{1}{x}}$$

$h=\frac{1}{x}$

$x\to\infty$

$h\to 0$

$$= 0\cdot \lim_{h\to 0}\frac{\sin h}{h}$$

$$= 0\cdot 1 = \boxed{0}$$

choose $x = 10000$

Radian Mode

Evaluate $\sqrt{10000}\cdot\sin\frac{1}{10000} \approx .01$

Let $x = 1000000$

$\sqrt{1000000}\cdot\sin\frac{1}{1000000} \approx 1E\text{-}3 = .001$

As $x\to\infty$, $\sqrt{x}\,\sin\frac{1}{x}\to 0$

Mar 19-9:32 AM

Evaluate

1) $\lim_{x\to\infty} \cos\frac{1}{x}$ $\left(\frac{1}{x}\to 0\right)$ $= \boxed{1}$

2) $\lim_{x\to\infty}\left(\sqrt{x^2+2ax}-\sqrt{x^2+2bx}\right) = \infty-\infty$ I.F.

$$= \lim_{x\to\infty} \frac{\left(\sqrt{x^2+2ax}-\sqrt{x^2+2bx}\right)\left(\sqrt{x^2+2ax}+\sqrt{x^2+2bx}\right)}{\sqrt{x^2+2ax}+\sqrt{x^2+2bx}}$$

$(A-B)(A+B)=A^2-B^2$

$$= \lim_{x\to\infty} \frac{\left(\sqrt{x^2+2ax}\right)^2-\left(\sqrt{x^2+2bx}\right)^2}{\sqrt{x^2+2ax}+\sqrt{x^2+2bx}}$$

$$= \lim_{x\to\infty} \frac{x^2+2ax-(x^2+2bx)}{\sqrt{x^2+2ax}+\sqrt{x^2+2bx}}$$

$$= \lim_{x\to\infty} \frac{2ax-2bx}{\sqrt{x^2+2ax}+\sqrt{x^2+2bx}} = \frac{\infty}{\infty}$$

as $x\to\infty$
$x=\sqrt{x^2}$

$$= \lim_{x\to\infty} \frac{\frac{2ax}{x}-\frac{2bx}{x}}{\sqrt{\frac{x^2}{x^2}+\frac{2ax}{x^2}}+\sqrt{\frac{x^2}{x^2}+\frac{2bx}{x^2}}}$$

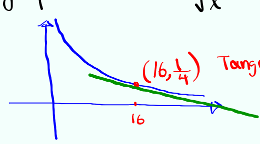
$$= \lim_{x\to\infty} \frac{2a-2b}{\sqrt{1+\frac{2a}{x}}+\sqrt{1+\frac{2b}{x}}} = \frac{2a-2b}{\sqrt{1}+\sqrt{1}}$$

$$= \frac{\cancel{2}(a-b)}{\cancel{2}} = \boxed{a-b}$$

Mar 19-9:42 AM

Find the equation of the tan. line to the graph of $f(x)=\frac{1}{\sqrt{x}}$ at $x=16$.

$f(16)=\frac{1}{\sqrt{16}}=\frac{1}{4}$



$m = f'(16) = \frac{-1}{2\cdot 16\sqrt{16}} = \boxed{\frac{-1}{128}}$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{\frac{1}{\sqrt{x+h}}-\frac{1}{\sqrt{x}}}{h}$$

LCD $=\sqrt{x+h}\sqrt{x}$

$$= \lim_{h\to 0}\frac{\sqrt{x}-\sqrt{x+h}}{h\sqrt{x+h}\sqrt{x}}$$

$$= \lim_{h\to 0}\frac{(\sqrt{x}-\sqrt{x+h})(\sqrt{x}+\sqrt{x+h})}{h\sqrt{x+h}\sqrt{x}(\sqrt{x}+\sqrt{x+h})}$$

$$= \lim_{h\to 0}\frac{(\sqrt{x})^2-(\sqrt{x+h})^2}{h\sqrt{x+h}\sqrt{x}(\sqrt{x}+\sqrt{x+h})}$$

$$= \lim_{h\to 0}\frac{-\cancel{h}^{1}}{\cancel{h}\sqrt{x+h}\sqrt{x}(\sqrt{x}+\sqrt{x+h})}$$

$$= \frac{-1}{\sqrt{x}\sqrt{x}(\sqrt{x}+\sqrt{x})} = \frac{-1}{x\cdot 2\sqrt{x}} = \boxed{\frac{-1}{2x\sqrt{x}}}$$

$y-y_1=m(x-x_1)$

$y-\frac{1}{4}=\frac{-1}{128}(x-16) \Rightarrow \boxed{y=\frac{-1}{128}x+\frac{3}{8}}$

Mar 19-9:54 AM

Prove $\frac{d}{dx}[x^4] = 4x^3$

$f(x) = x^4$ $\qquad \frac{d}{dx}[f(x)] = f'(x)$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Review Binomial theorem

$$= \lim_{h \to 0} \frac{(x+h)^4 - x^4}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^4} + 4x^3h + 6x^2h^2 + 4xh^3 + h^4 \cancel{- x^4}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(4x^3 + \underbrace{6x^2h}_{\to 0} + \underbrace{4xh^2}_{\to 0} + \underbrace{h^3}_{\to 0})}{\cancel{h}} = \boxed{4x^3}$$

Mar 19-10:08 AM

Differentian Rules

1) $\frac{d}{dx}[c] = 0$

2) $\frac{d}{dx}[x] = 1$

3) $\frac{d}{dx}[x^n] = n\,x^{n-1}$

4) $\frac{d}{dx}[c\,f(x)] = c\,\frac{d}{dx}[f(x)]$

5) $\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}[f(x)] + \frac{d}{dx}[g(x)]$

6) $\frac{d}{dx}[f(x) - g(x)] = \frac{d}{dx}[f(x)] - \frac{d}{dx}[g(x)]$

Mar 19-10:14 AM

Find $f'(x)$ if $f(x) = x^3 - 2x^2 + 5$

$$f'(x) = \frac{d}{dx}[f(x)]$$
$$= \frac{d}{dx}[x^3 - 2x^2 + 5]$$
$$= \frac{d}{dx}[x^3] - \frac{d}{dx}[2x^2] + \frac{d}{dx}[5]$$
$$= 3x^{3-1} - 2\frac{d}{dx}[x^2] + 0$$
$$= 3x^2 - 2 \cdot 2x^{2-1} = \boxed{3x^2 - 4x}$$

Mar 19-10:21 AM

Find $f'(x)$ for $f(x) = \sqrt{x}$, then find $f(4)$ & $f'(4)$

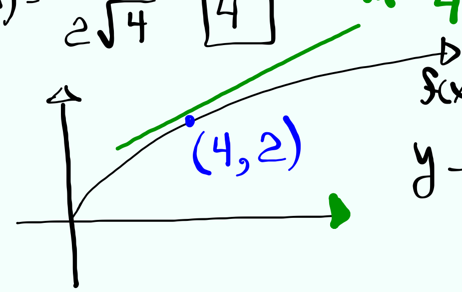
$$\frac{d}{dx}[x^n] = nx^{n-1}$$

$$f(x) = x^{1/2}$$
$$f'(x) = \frac{d}{dx}[x^{1/2}]$$
$$= \frac{1}{2} \cdot x^{\frac{1}{2}-1}$$

$$f(4) = \sqrt{4} = \boxed{2}$$

$$f'(4) = \frac{1}{2\sqrt{4}} = \boxed{\frac{1}{4}}$$

$m = \frac{1}{4}$



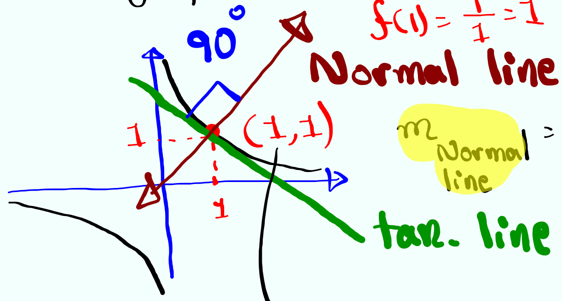
$$f(x) = \sqrt{x} \quad = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2x^{1/2}} = \frac{1}{2\sqrt{x}}$$

$$y - 2 = \frac{1}{4}(x - 4)$$
$$\vdots$$
$$\boxed{y = \frac{1}{4}x + 1}$$

Mar 19-10:26 AM

Find equation of the normal line to the graph of $f(x)=\frac{1}{x}$ at $x=1$.

$f(1)=\frac{1}{1}=1$



$$m_{\text{Normal line}} = \frac{-1}{m_{\text{tangent line}}} = \frac{-1}{f'(1)} = \frac{-1}{\frac{-1}{(1)^2}} = \boxed{1}$$

$$f(x)=\frac{1}{x}=x^{-1}$$

$$f'(x)=\frac{d}{dx}\left[x^{-1}\right] = -1x^{-1-1} = -1x^{-2} = \frac{-1}{x^2}$$

$$y-y_1=m(x-x_1)$$

$$y-1=1(x-1) \longrightarrow \boxed{y=x}$$

Mar 19-10:33 AM

Find all x-values where $f'(x)=0$ for $f(x)=\frac{1}{2}x^6-3x^4+1$.

Solve $f'(x)=0$

$$f'(x)=\frac{d}{dx}\left[\tfrac{1}{2}x^6-3x^4+1\right]$$

$$=\frac{d}{dx}\left[\tfrac{1}{2}x^6\right]-\frac{d}{dx}\left[3x^4\right]+\frac{d}{dx}[1]$$

$$=\frac{1}{2}\frac{d}{dx}\left[x^6\right]-3\frac{d}{dx}\left[x^4\right]+\frac{d}{dx}[1]$$

$$=\frac{1}{2}\cdot 6x^5-3\cdot 4x^3+0$$

$$=3x^5-12x^3$$

$f'(x)=0$

$3x^5-12x^3=0$

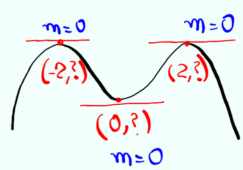
$3x^3(x^2-4)=0$

$3x^3(x+2)(x-2)=0$

$x^3=0$ $\boxed{x=0}$

$x+2=0$ $\boxed{x=-2}$

$x-2=0$ $\boxed{x=2}$



Why $m=0$?

$f'(x)=0$

Mar 19-10:42 AM

Prove $\frac{d}{dx}[\sin x] = \cos x$

$f(x) = \sin x$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h}$$

$$= \lim_{h \to 0} \frac{\boxed{\sin x \cosh} + \cos x \sin h \boxed{- \sin x}}{h}$$

$$= \lim_{h \to 0} \frac{\sin x[\cos h - 1]}{h} + \lim_{h \to 0} \frac{\cos x \sin h}{h}$$

$$= \sin x \underbrace{\lim_{h \to 0} \frac{\cos h - 1}{h}}_{\to 0} + \cos x \underbrace{\lim_{h \to 0} \frac{\sin h}{h}}_{\to 1}$$

$$= \sin x \cdot 0 + \cos x \cdot 1 = 0 + \cos x = \boxed{\cos x}$$

$$\boxed{\frac{d}{dx}[\sin x] = \cos x} \qquad \boxed{\frac{d}{dx}[\cos x] = -\sin x}$$

Mar 19-10:52 AM

Class Quiz 11 $\boxed{\text{Open Notes}}$

Find equation of the tangent line to the graph of $f(x) = x^4 + 2x^2 - x$ at $x = 1$ in Slope-Int. form.

$f(1) = 1^4 + 2(1)^2 - 1 = 1 + 2 - 1 = \boxed{2}$

$f'(x) = 4x^3 + 2 \cdot 2x - 1$

$f'(x) = 4x^3 + 4x - 1$

$f'(1) = 4(1)^3 + 4(1) - 1 = \boxed{7}$

$y - y_1 = m(x - x_1)$

$y - 2 = 7(x - 1)$

$y - 2 = 7x - 7$

$\boxed{y = 7x - 5}$

Mar 19-11:00 AM